EECE 301 NETWORK ANALYSIS II Dr. Charles Kim

Class Note 12: Summary on Power Definition and Power Factor

A. POWERS

When a load is connected to a source, it consumes (or absorbs) 'power'. In some cases, it could deliver 'power' to the source. It all depends on 'which power' we are talking about. Let's have definitions and some observations.

1. Definitions

Instantaneous Power: $p(t) = v(t) \cdot i(t) = P\cos\theta + P\cos 2wt - Q\sin 2wt$

where $\theta = \theta_v - \theta_i$ (angle difference between voltage and current)

Real Power: $P = \frac{V_m I_m}{2}\cos\theta = V_{rms} I_{rms} \cos\theta$

Reactive Power: $Q = \frac{V_m I_m}{2}\sin\theta = V_{rms} I_{rms} \sin\theta$

Complex Power: $\bar{S} = P + jQ$ (*note: see #3 for deeper discussion)

Apparent Power: $|S| = \sqrt{P^2 + Q^2}$

Power Factor Angle: $\theta = \theta_v - \theta_i$ (phase angle difference between voltage and current)

2. Load Dependence of the Power

Pure R case: $\theta = 0$, $P = \frac{V_m I_m}{2}$, $Q = 0$ therefore, "R consumes P only"

Pure L case: $\theta = 90^\circ$, $P = 0$, $Q = \frac{V_m I_m}{2}$, therefore, "L consumes Q only"

Pure C case: $\theta = -90^\circ$, $P = 0$, $Q = -\frac{V_m I_m}{2}$, therefore, "C ***delivers*** Q " toward the source!!!!!

General R, L, and C case?

See the discussions next page

NOTE: Do you remember the "polarity of power" thing? If not, check with note01 of *Network Analysis I* at my web site. It deals with power at the source's point of view. But it can be applied to the load's point of view.

Bottom Line: *negative power delivers*; *positive power consumes*.

3. Complex Power in Phasor

From $\bar{S} = P + jQ$,

$\bar{S} = V_{rms}I_{rms}\cos\theta + jV_{rms}I_{rms}\sin\theta = V_{rms}I_{rms}(\cos\theta + j\sin\theta) = V_{rms}I_{rms}e^{j\theta}$

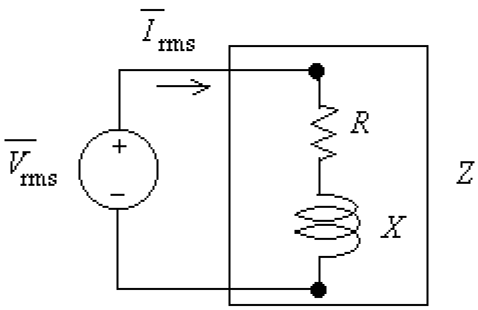
Since $\theta = \theta_v - \theta_i$

$\bar{S} = V_{rms}I_{rms}e^{j\theta} = V_{rms}I_{rms}\angle\theta = V_{rms}I_{rms}\angle(\theta_v - \theta_i) = V_{rms}I_{rms}\angle\theta_v \cdot \angle-\theta_i = V_{rms}\angle\theta_v \cdot I_{rms}\angle-\theta_i$

Therefore,

$\bar{S} = \bar{V}_{rms} \cdot \bar{I}^*{}_{rms}$

Let's expand the complex Power S. See an example circuit below for the discussion.



(1) From $\bar{V}_{rms} = Z \cdot \bar{I}_{rms}$,

$\bar{S} = \bar{V}_{rms} \cdot \bar{I}^*{}_{rms} = \bar{Z}\bar{I}_{rms}\bar{I}^*{}_{rms} = \bar{Z}I^2_{rms} = I^2_{rms}(R + jX) = R \cdot I^2_{rms} + jX \cdot I^2_{rms}$

In this case: $P = R \cdot I^2_{rms}$ and $Q = X \cdot I^2_{rms}$

In other words, ***P relates only to R, and Q, only to X.***

(2) Alternatively, from $\bar{I}_{rms} = \frac{\bar{V}_{rms}}{\bar{Z}}$, $\bar{S} = \bar{V}_{rms} \cdot [\frac{\bar{V}_{rms}}{\bar{Z}}]^* = \frac{V^2_{rms}}{\bar{Z}^*} = \frac{V^2_{rms}}{R - jX} = \frac{V^2_{rms}(R + jX)}{R^2 + X^2}$

By expanding further, $\bar{S} = \frac{V^2_{rms}(R + jX)}{R^2 + X^2} = \frac{V^2_{rms}(R + jX)}{Z^2} = \frac{V^2_{rms}R}{Z^2} + j\frac{V^2_{rms}X}{Z^2}$

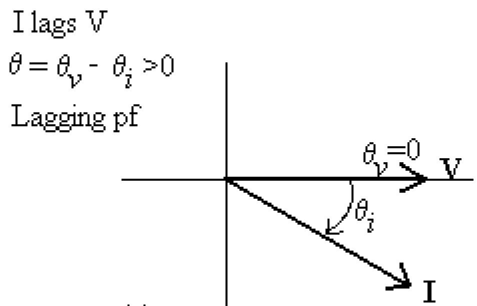
In this case: $P = \frac{V^2_{rms}R}{Z^2}$ and $Q = \frac{V^2_{rms}X}{Z^2}$

In other words, again, ***P relates only to R, and Q, only to X.***

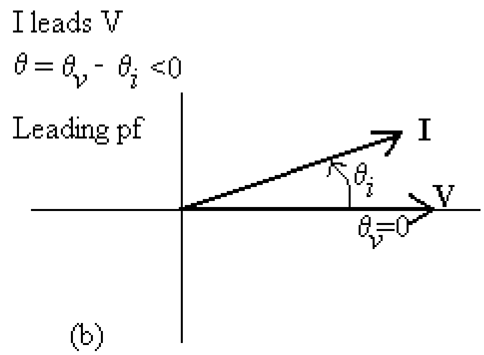
B. POWER FACTOR (*pf*)

1. Definition

Power factor (pf) is defined by "cosine of the angle made by voltage and current." The angle, θ, is defined by $\theta_v - \theta_i$, where θ_v and θ_i are phase angles of the voltage and the current, respectively. This definition indirectly says that the reference in phase domain is the voltage. (See the diagrams below)



(a)



(b)

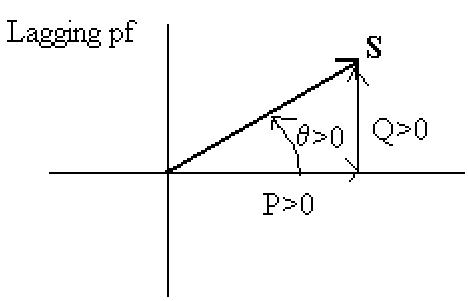
"Leading pf" means that the current leads the voltage and therefore the pf angle θ is negative, as shown in (b); "Lagging pf" means that the current lags the voltage and the pf angle θ is positive as in (a).

2. Alternative Definition

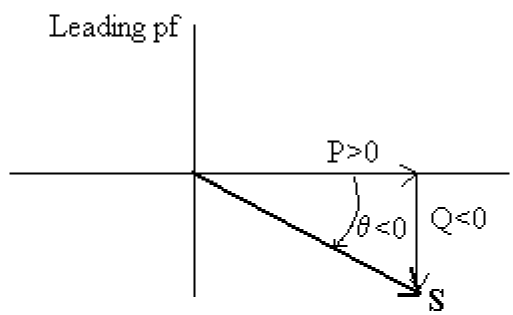
Let's expand the original definition of the pf to the complex power ***S***. Complex power is defined:

$$S = \overline{VI}^* = V\angle\theta_v \cdot I\angle -\theta_i = VI\angle(\theta_v - \theta_i) = VI\angle\theta$$

Therefore the angle of the complex power is exactly same as the angle of V and I. In other words, if we know the complex power and present it on the complex plane, we get the power factor angle. Also, since $S = P + jQ$, power factor angle can also be found once we know P and Q. (See diagrams below) Or, we can draw the general place of a complex power, once we know power factor is leading or lagging. "leading" or "lagging" power factor determines the polarity of reactive power Q.



(a)



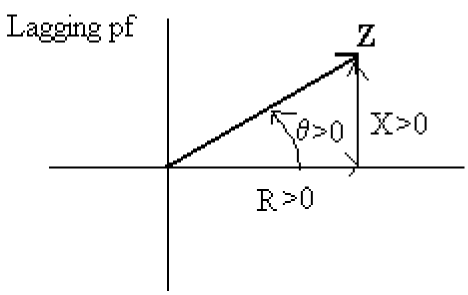
(b)

3. Another Alternative Definition

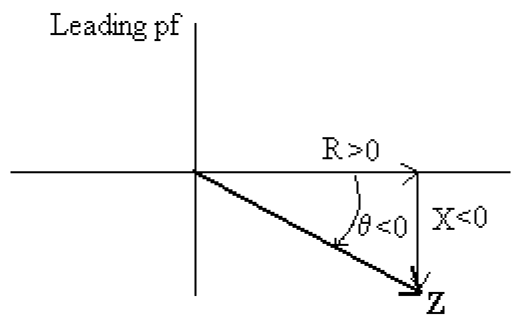
Let's play a little bit more. This time we will involve load impedance Z. Since load voltage V and load current I determines the load impedance Z, i.e., Z=V/I, we can express the power factor with Z.

$$Z = \frac{\bar{V}}{\bar{I}} = \frac{V\angle\theta_v}{I\angle\theta_i} = \frac{V}{I}\angle(\theta_v - \theta_i) = \frac{V}{I}\angle\theta$$

Therefore, if we locate the load impedance Z, the angle made by the impedance is the power factor angle. Since the impedance is composed of resistance R and reactance X, $Z = R + jX$, we can relate these elements with power factor as shown below.



(a)



(b)

4. Final Words

If you compare the phase diagram of S and Z, you may notice that they are places along the same line, either they are lagging or leading. In many problems of power factor calculation, you may want to apply any or all of the definitions presented here. In any power factor related problems, I recommend you draw phase diagram of **S**, **Z**, or **V** & **I**, before you jump to write your answer. Always careful with the words "leading" and "lagging."