EECE 301 Network Analysis II Dr. Charles Kim

Class Note 10: Steady-State Sinusoidal Response and Phasor Transformation

A. Steady-State Sinusoidal Response---S-domain perspective

1. Sinusoidal Source

(i) Periodic (*cosine* or *sine*) function with period T, where $T = 2\pi / w$.

(ii)Expressed by: Max (or peak) value, Angular Speed (w), and Phase angle(ϕ)

$$v(t) = V_m \cos(wt + \phi)$$

(iii) "Average", "Mean-Square", and "Root-Mean-Square (RMS)" values:

Time Average (or "DC equivalent" or "DC" value): $\overline{v(t)} = \frac{1}{T}\int_{t_o}^{t_o+T} v(t)dt$

Mean-Square Value (or "Normalized Power): $\overline{v^2(t)} = \frac{1}{T}\int_{t_o}^{t_o+T} v^2(t)dt$

RMS (or "Effective Value"): $v(t)_{RMS} = \sqrt{\frac{1}{T}\int_{t_0}^{t_o+T} v^2(t)dt}$

2. Steady-State Response of a sinusoidal source---S-Domain Perspective

(i) There could be another discussion on this in differential equation perspective

(ii) Since we just passed through s-domain and transfer function, we see in s-domain's perspective.

(iii)From the discussion of steady-state sinusoidal response, we had the following formula:

$$y(t) = A|H(jw)|\cos[wt + \phi + \theta(w)],$$

where, input is given with $x(t) = A\cos[wt + \phi]$

|H(jw)| is the magnitude of the steady-state transfer function of a circuit.

and, θ(w) is the phase angle of the steady-state transfer function H(jw)

(iv) If we compare the input and the output (response), the input frequency is kept intact, while the magnitude and phase angle are changed.

(v) Also, the magnitude and the phase angle change is determined by the steady-state transfer function of a circuit, and the transfer function is also determined by the circuit elements, (R, L, and C)

(vi) Therefore, if we ignore the frequency (which does not change), we can express the sinusoidal inputs and outputs only with magnitudes and phase angles.---> This is the basic idea behind the Phasor Transformation.

B. Phasor Transformation

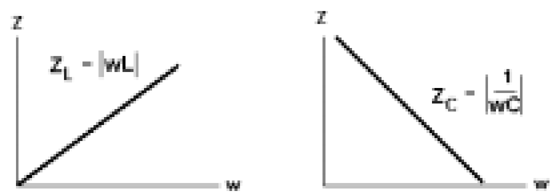
1. “Phasor”: Complex number representation (with Amplitude & Angle) of a sinusoid.

$v(t) = V_m \cos(wt + \phi)$ ---------***P***---------> $\overline{V} = V_m e^{i\phi} = V_m\angle\phi$

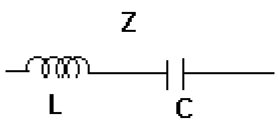
2. Phasor Representation of Circuit Element

Element	Time Domain	Phasor Domain
Resistor	R	R
Inductor	L	jwL
Capacitor	C	$\frac{1}{jwC}$ or $\frac{-j}{wC}$

3. Impedance Dependency on angular frequency (w)



4. Resonance: How to make the impedance of LC series zero? At what value of *w*?



Let's write an impedance equation: $Z = jwL - j\frac{1}{wC} = j(wL - \frac{1}{wC})$

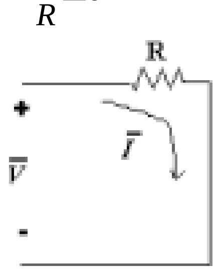
The impedance Z becomes zero when $wL - \frac{1}{wC} = 0$, i.e. $w = \frac{1}{\sqrt{LC}}$

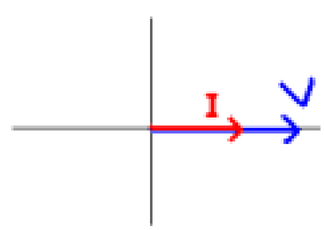
5. Circuit Analysis with Phasor: the order

Step 1: Convert a circuit into a phasor domain circuit
Step 2: Simplification and Analysis of the phasor circuit
Step 3: Solution and Response
Step 4: Inverse Phasor transformation to convert the response back to time domain

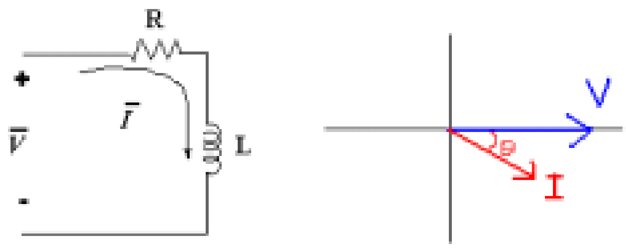
6. Phasor Diagram:

(i). R only: $\bar{I} = \frac{\bar{V}}{R} = \frac{V}{R}\angle 0^{\circ}$

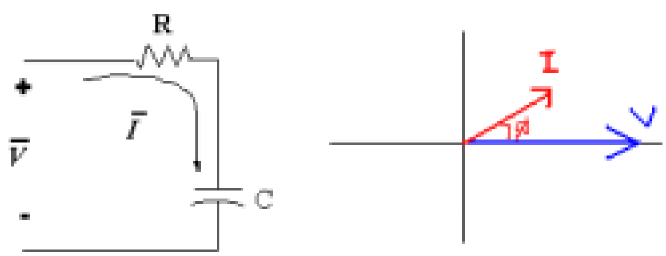




(ii). R-L only: $\bar{I} = \dfrac{\bar{V}}{R + jwL} = \dfrac{V\angle 0}{\sqrt{R^2 + w^2L^2}\angle\theta} = \dfrac{V}{\sqrt{R^2 + w^2L^2}}\angle -\theta$

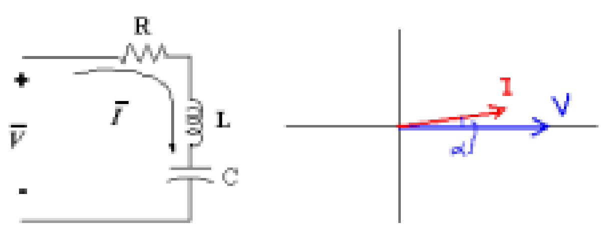


(iii). R-C only: $\bar{I} = \dfrac{\bar{V}}{R - j/wC} = \dfrac{V\angle 0}{\sqrt{R^2 + 1/w^2C^2}\angle -\phi} = \dfrac{V}{\sqrt{R^2 + 1/w^2C^2}}\angle\phi$



(iv). R-L-C case: $\bar{I} = \dfrac{\bar{V}}{R + jX_L - jX_c} = \dfrac{V\angle 0}{\sqrt{R^2 + (X_L - X_C)^2}\angle -\alpha} = \dfrac{V}{\sqrt{R^2 + (X_L - X_C)^2}}\angle\alpha$

where $X_L = wL$ and $X_c = 1/wC$



C. Trigonometry Formula

$\sin(A+B) = \sin A\cos B + \cos A\sin B$ $\quad$ $\sin(A-B) = \sin A\cos B - \cos A\sin B$

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$\sin A\sin B = \dfrac{1}{2}(\cos(A-B) - \cos(A+B))$ $\quad$ $\cos A\cos B = \dfrac{1}{2}(\cos(A-B) + \cos(A+B))$

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$A\cos\alpha + B\sin\alpha = \sqrt{A^2 + B^2}\cos(\alpha - \theta)$, where $\theta = \arctan\dfrac{B}{A}$

$$\cos^2 A = \frac{1+\cos 2A}{2} \qquad \sin^2 A = \frac{1-\cos 2A}{2}$$

$$\sin 2A = 2\sin A\cos A = \frac{2\tan A}{1+\tan^2 A}$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \frac{1-\tan^2 A}{1+\tan^2 A}$$

D. Complex Number -revisited

1. Complex Number Notation:

Rectangular form: $x = a + jb$

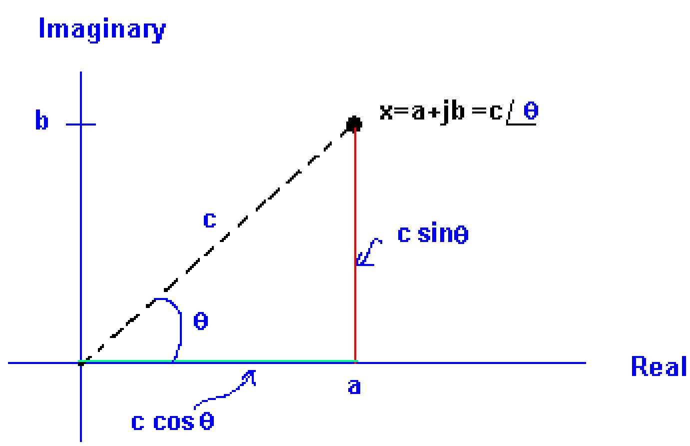
Polar form: $x = c \cdot e^{j\theta} = c\angle\theta$ where $c = \sqrt{a^2 + b^2}$ and $\theta = \tan^{-1}\frac{b}{a}$

2. Euler's Identity – Exponential relation

$$e^{j\theta} = \cos\theta + j\sin\theta$$
$$e^{-j\theta} = \cos\theta - j\sin\theta$$

Then, $x = a + jb = \sqrt{a^2+b^2}e^{j\theta} = \sqrt{a^2+b^2}\cdot(\cos\theta + j\sin\theta)$,

From above, $a = \sqrt{a^2+b^2}\cdot\cos\theta$ and $b = \sqrt{a^2+b^2}\cdot\sin\theta$



3. Arithmetic Operations

$x = 8 + j10$, $y = 5 - j4$

Addition and Subtraction: *Using rectangular form is easier*

$x + y = (8 + j10) + (5 - j4) = 13 + j6$

$x - y = (8 + j10) - (5 - j4) = 3 + j16$

Multiplication: *Polar form is usually simpler*

From $x = 8 + j10 = 12.81\angle 51.34$ and $y = 5 - j4 = 6.40\angle -38.66$

$$xy = (12.81\angle 51.34)(6.40\angle -38.66) = (12.81 \times 6.40)\angle(51.34 - 38.66) = 82\angle 12.68$$

(*magnitude multiplication & angle addition*)

Division: *Polar form is usually simpler* (In rectangular, multiply the numerator and the denominator by the conjugate of the denominator)

$$\frac{x}{y} = \frac{12.81\angle 51.34}{6.40\angle -38.66} = \frac{12.81}{6.40}\angle(51.34 + 38.66) = 2\angle 90 = 0 + j2 = j2$$

(*magnitude division & angle subtraction*)

4. Final Note: Refer Appendix B of the book for an extensive review