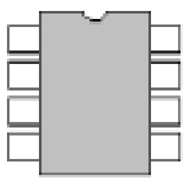
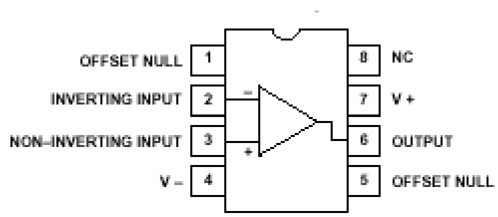
## Note09: Active Filters ---Part 1

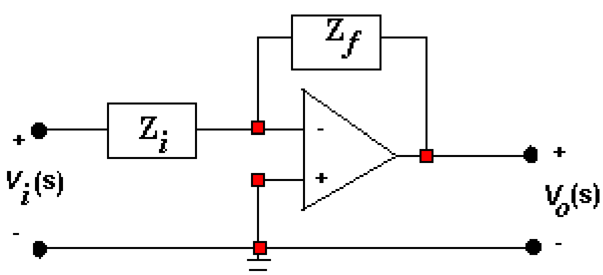
1. Active circuits employing OP Amps, resistors, and capacitors
2. Amplification gain
3. No load effect (Remember in passive filters with load at the filter terminals?)
4. Topics
   - (i) Simple LPF
   - (ii)Simple HPF
   - (iii)Prototype Filter and Scaling Concept
   - (iv)Higher Order Filter
     - * Cascading Identical Filters
     - * Butterworth Filters






0. General First-Order Active Filter

Circuit Configuration:



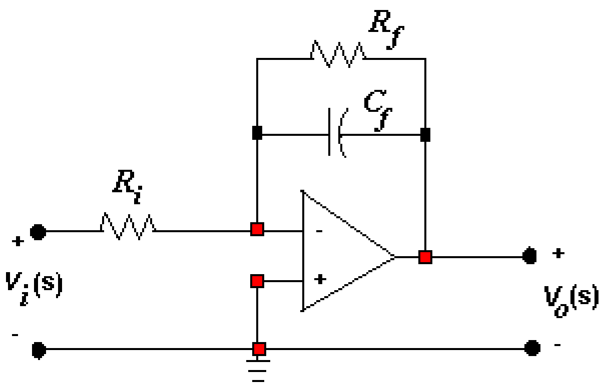

Node voltage equation at the (-) node: Remember that there is no current to the Op-Amp.

$$\frac{0-V_i}{Z_i} = \frac{0-V_o}{Z_f} \quad \text{----->} V_o = -V_i \cdot \frac{Z_f}{Z_i}$$

s-domain Transfer Function: $H(s) = \dfrac{V_o(s)}{V_i(s)} = \dfrac{-V_i(s)\dfrac{Z_f}{Z_i}}{V_i(s)} = -\dfrac{Z_f}{Z_i}$

1. First-Order Low Pass Filter

Circuit Configuration:




s-domain Transfer Function:

$$Z_i = R_i \quad \text{and} \qquad Z_f = R_f \,//\, C_f \Rightarrow \frac{\dfrac{R_f}{sC_f}}{R_f + \dfrac{1}{sC_f}}$$

Therefore,

$$H(s) = -\frac{Z_f}{Z_i} = -\frac{\dfrac{\dfrac{R_f}{sC_f}}{R_f + \dfrac{1}{sC_f}}}{R_i} = \frac{R_f/sC_f}{R_i(R_f + 1/sC_f)} = -\frac{R_f}{R_i(sR_fC_f + 1)} = -\frac{R_f}{R_i} \cdot \frac{1}{(1 + R_fC_fs)}$$

Steady-State Transfer Function: $H(jw) = -\dfrac{R_f}{R_i} \cdot \dfrac{1}{1 + jR_fC_fw}$

**Pass Band Gain:** $A(0) = -\dfrac{R_f}{R_i}$

Relative dB Amplitude:

$$A_{dB}(w) = 20\log(R_f/R_i) - 10\log[1 + \left(\frac{w}{1/R_fC_f}\right)^2]$$

$$= 20\log(R_f/R_i) - 10\log[1 + \left(\frac{w}{w_c}\right)^2]$$

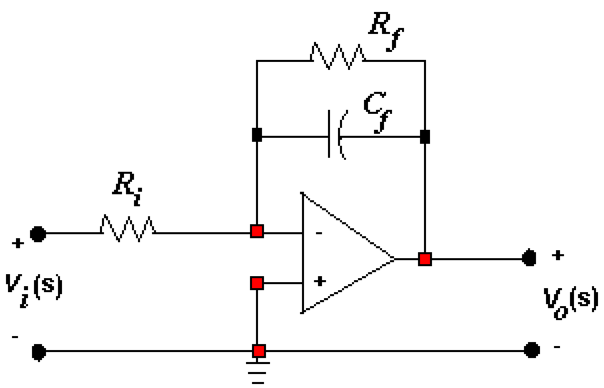
where $w_c$= cutoff frequency in [rad/sec]

Bode-Plot:

What is the cut-off frequency? Is it $\dfrac{w_c}{1/R_fC_f} = 1.0$ ?

What is the cut-off frequency and passband gain with $R_i$=400, $R_f$=1600, $C_f$=0.2uF ?

**Example Problem:** Using the circuit below, design a low-pass filter with a pass-band gain of 15 dB and a cut-off frequency of 10 kHz. Assume a 5 nF capacitor is available.
(a) Specify the values of the resistors.
(b) Draw a Bode Plot




**Solution**

Given Information:

(1) Pass band gain (i.e. $G_{dB}(0)$=15dB)
(2)$f_c$=10000 (i.e., $w_c=2\pi(10000)$ rad/s)
(3) C=5nF

What's the question, then? -->**Find $R_1$ and $R_2$.**

**SOLUTION:**

Bode Plot:

2. Scaling

1. Drawing a transfer function would be much easier with values of 1Ω, 1F, and 1H than with realistic values of elements.

2. From the LPF, the transfer function of $H(jw) = -\frac{R_f}{R_i} \cdot \frac{1}{1 + jR_f C_f w}$, can then become $H(jw) = -\cdot\frac{1}{1+w}$. And the cutoff frequency is $\frac{1}{R_2 C} = \frac{1}{1 \cdot 1} = 1$ [rad/sec]. This filter with all convenient values with $w_c$=1 [rad/sec] is called a ***prototype filter***.

3. After making computation for a transfer function using the convenient values of R, L, and C, we can transform the convenient values to realistic values using the process known as ***scaling***.

4. Two types of scaling:

(a) magnitude scaling ( $k_m$ : scale factor for magnitude)

R, L, C: convenient values
R', L', C': actual (realistic values)
Then, $R' = k_m R$, $L' = k_m L$, and $C' = C/k_m$

(b) frequency scaling ( $k_f$ : scale factor for frequency)

Then, $k_f = k_w = \frac{w_c'}{w_c}$, $R' = R$, $L' = L/k_f$, and $C' = C/k_f$

(since $wL = w'L'$ and $\frac{1}{wC} = \frac{1}{w'C'}$)

(c)Combination of two scale factors:

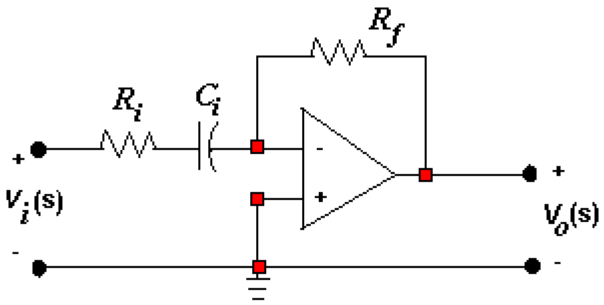
$$R' = k_m R, \quad L' = \frac{k_m}{k_f} L, \text{ and } C' = \frac{1}{k_m k_f} C$$

5. **Scaling Example**: use the prototype active LPF, along with magnitude and frequency scaling, to compute the resistor values with a gain of 5, a cutoff frequency of 1000 Hz, and a feedback capacitor of 0.01 uF.

Solution:

3. First Order High Pass Filter

Circuit Configuration:




$Z_i = R_i + 1/sC_i$ and $Z_f = R_f$

s-domain Transfer Function:

$$H(s) = -\frac{Z_f}{Z_i} = -\frac{R_f}{R_i + 1/sC_i} = -\frac{R_f}{R_i}\cdot\frac{1}{(1+1/R_iC_is)} = -\frac{R_f}{R_i}\cdot\frac{R_iC_is}{(1+R_iC_is)} = -\frac{R_f}{R_i}\cdot\frac{\frac{s}{1/R_iC_i}}{[1+\frac{s}{1/R_iC_i}]}$$

Steady-State: $H(jw) = -\frac{R_f}{R_i}\cdot\frac{j\frac{w}{1/R_iC_i}}{1+j\frac{w}{1/R_iC_i}} = -\frac{R_f}{R_i}\cdot\frac{j\frac{w}{w_c}}{1+j\frac{w}{w_c}}$, with $w_c = \frac{1}{R_iC_i}$.

**Pass band Gain:** $A(\infty) = -\frac{R_f}{R_i}$

Relative dB Amplitude: $A_{dB}(w) = 20\log\left(R_f/R_i\right) + 20\log[\left(\frac{w}{1/R_iC_i}\right)] - 10\log[1+\left(\frac{w}{1/R_iC_i}\right)^2]$

What is the cut-off frequency? Is it $\frac{w_c}{1/R_iC_i} = 1.0$?

What is the cut-off frequency and DC gain with $R_i$=800, $R_f$=4000, and $C_i$=0.1uF?