

Class Note 08: Passive Filters

A. Filter Introduction

Filter: A circuit that is designed to pass signals with desired frequencies and reject or attenuate others

Passive Filter: A filter consists of only passive elements such as R, L, and C

- Characteristics:*
1. Gain is less than 1
 2. May require bulky and expensive inductors
 3. Perform poorly at frequencies below the audio frequency
 4. Works fine at high frequencies

Active Filter: A filter consists of active elements such as transistors and OP Amps in addition to passive elements.

- Characteristics:*
1. Amplifier Gain
 2. No inductor is required
 3. Smaller and less expensive
 4. Less reliable and less stable
 5. Works fine at low frequencies (below 100 KHz)

Order of filters: Number of poles in the transfer function $G(s)$

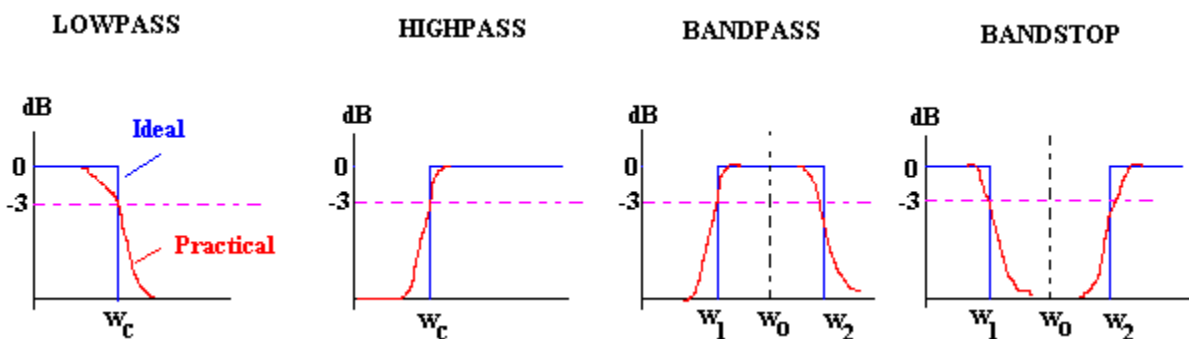
Four types of filters:

Low Pass Filter: $A_{dB}(0)=0dB$, $A_{dB}(\infty)=-\infty$ dB, $A_{dB}(w_c)=-3dB$

High Pass Filter: $A_{dB}(0)=-\infty dB$, $A_{dB}(\infty)=0dB$, $A_{dB}(w_c)=-3dB$

Band Pass Filter: $A_{dB}(0)=-\infty dB$, $A_{dB}(\infty)=-\infty$ dB, $A_{dB}(w_c)=0dB$

Band Stop (or Band Reject or Notch) Filter: $A_{dB}(0)=0dB$, $A_{dB}(\infty)=0dB$, $A_{dB}(w_0)=0dB$



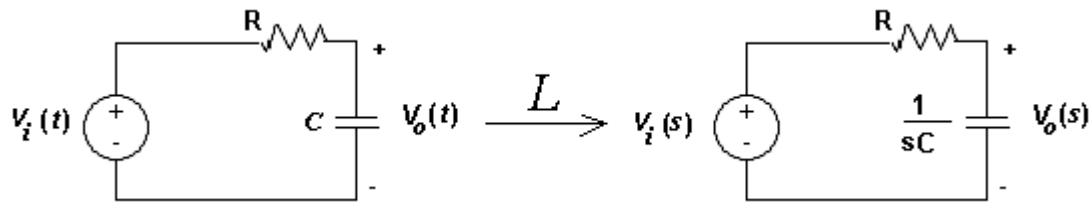
NOTE: Do you see that relative $3dB = \frac{1}{\sqrt{2}}$ (absolute) ?

Is this an RMS value for a max value=1?

B. Passive Filters

1. Low Pass Filter with series RC

Circuit Configuration:



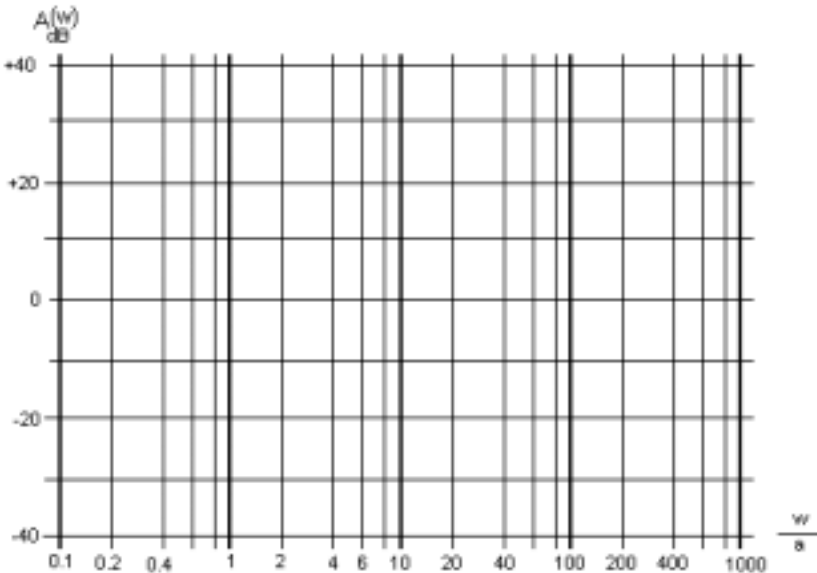
s-domain Transfer Function:

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{V_i(s) \left(\frac{1/sC}{R + 1/sC} \right)}{V_i(s)} = \frac{1/sC}{R + 1/sC} = \frac{1}{1 + RCs} = \frac{1}{1 + \frac{s}{1/RC}}$$

Steady-State Transfer Function: $H(j\omega) = \frac{1}{1 + j \frac{\omega}{1/RC}}$

Relative dB Amplitude: $A_{dB}(\omega) = 20 \log \frac{1}{\sqrt{1 + \left(\frac{\omega}{1/RC} \right)^2}} = -10 \log \left[1 + \left(\frac{\omega}{1/RC} \right)^2 \right]$

Bode-Plot:



Do you see that the cut-off frequency (at -3dB) is $\frac{1}{RC}$? or $\omega_c RC = 1.0$

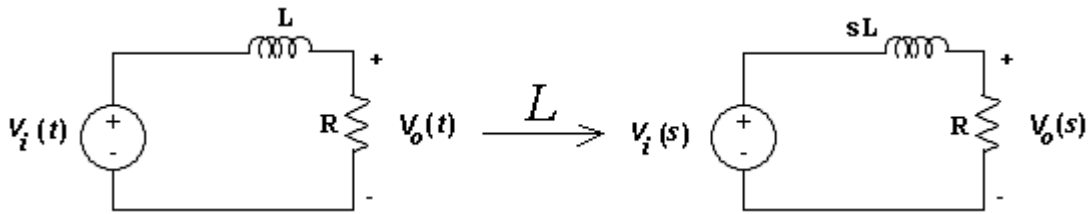
Can you choose the resistor R that will yield a low pass filter with a cutoff frequency of 3 KHz, if there is only 1uF capacitors?

Response Example: If the input is $v_i(t) = 480 \cos \omega t$, write the steady-state expression for the response $v_o(t)$ at $\omega = \omega_c$, $0.2\omega_c$, and $8\omega_c$. (assume that $R=20000$ and $C=4$ nF)

(SOLUTION)

2. Low Pass Filter with series RL

Circuit Configuration:

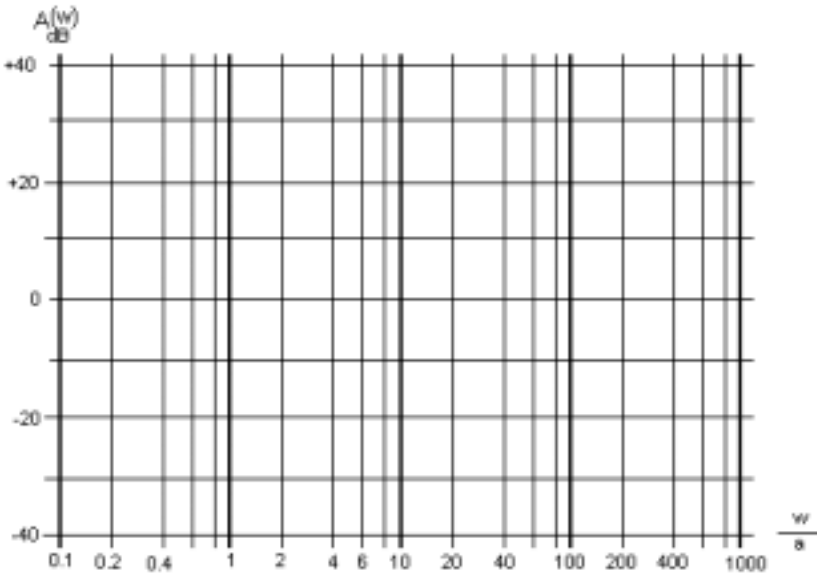


$$\text{s-domain Transfer Function: } H(s) = \frac{V_o(s)}{V_i(s)} = \frac{V_i(s) \left(\frac{R}{R + sL} \right)}{V_i(s)} = \frac{R}{R + sL} = \frac{1}{1 + sL/R} = \frac{1}{1 + \frac{s}{R/L}}$$

$$\text{Steady-State Transfer Function: } H(j\omega) = \frac{1}{1 + j \frac{\omega}{R/L}}$$

$$\text{Relative dB Amplitude: } A_{dB}(\omega) = 20 \log \frac{1}{\sqrt{1 + \left(\frac{\omega}{R/L} \right)^2}} = -10 \log \left[1 + \left(\frac{\omega}{R/L} \right)^2 \right]$$

Bode-Plot:



Do you see that the cut-off frequency (at -3dB) is $\frac{R}{L}$? or $\frac{\omega_c}{R/L} = 1.0$

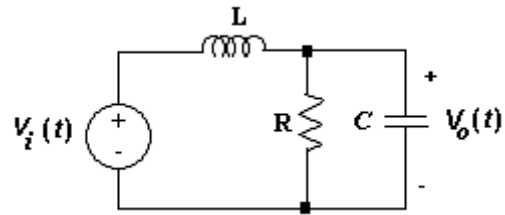
What is the cut-off frequency with $R=6.3 \Omega$ and $L=100 \text{ mH}$?

Response Example: If the input is $v_i(t) = 480 \cos 1000t$, find the magnitude of the output voltage at $\omega = \omega_c$, $0.2\omega_c$, and $8\omega_c$. (assume that $R=1500$ and $L=0.25H$)

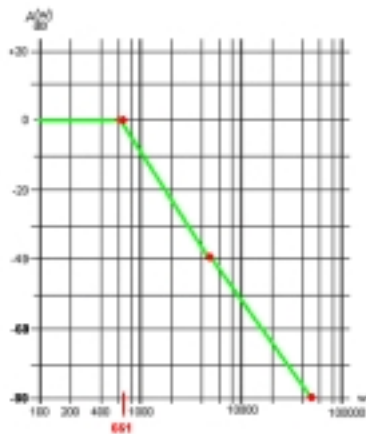
(SOLUTION)

Example Problem: Determine the filter type of the circuit below and draw a Bode plot, and find the cutoff frequency.

($R=2K$, $L=2$ H, and $C=2\mu F$)

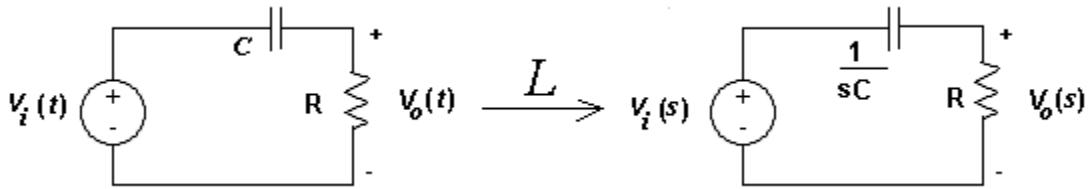


SOLUTION



3. High Pass Filter with series RC

Circuit Configuration:



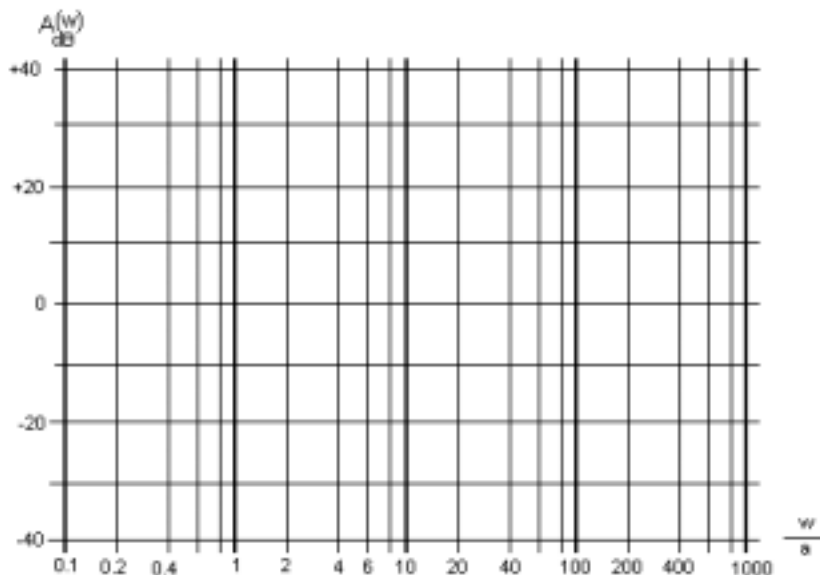
$$\text{s-domain Transfer Function: } H(s) = \frac{V_o(s)}{V_i(s)} = \frac{V_i(s) \left(\frac{R}{R + 1/sC} \right)}{V_i(s)} = \frac{R}{R + 1/sC} = \frac{\frac{s}{1/RC}}{1 + \frac{s}{1/RC}}$$

$$\text{Steady-State Transfer Function: } H(j\omega) = \frac{j \frac{\omega}{1/RC}}{1 + j \frac{\omega}{1/RC}}$$

Relative dB Amplitude:

$$A_{dB}(\omega) = 20 \log \frac{\left(\frac{\omega}{1/RC} \right)}{\sqrt{1 + \left(\frac{\omega}{1/RC} \right)^2}} = 20 \log \left[\left(\frac{\omega}{1/RC} \right) \right] - 10 \log \left[1 + \left(\frac{\omega}{1/RC} \right)^2 \right]$$

Bode-Plot:



Do you see that the cut-off frequency (at -3dB) is $\frac{1}{RC}$? or $\omega_c RC = 1.0$

Can you find the cutoff frequencies for the following resistor values:
 100Ω , 5000Ω , and 30000Ω , if there is only $1\mu\text{F}$ capacitors?