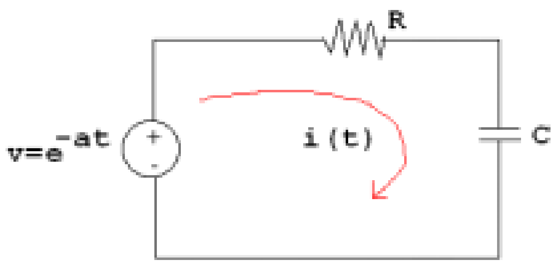
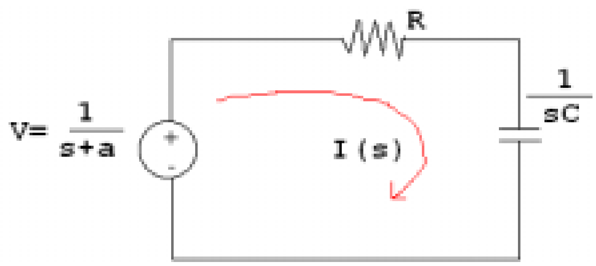
Class Note 04: More on Laplace Transformation Application to Circuit Analysis
SOLUTION
1. Exponential Source Case



Assuming that there is no initial condition, for t>0, we have the following s-domain circuit.

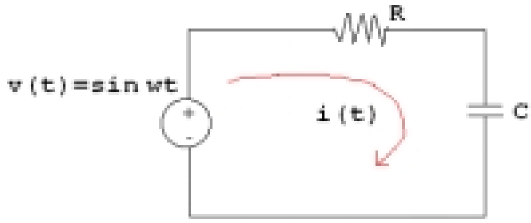


Then, the s-domain equation for I(s) is:

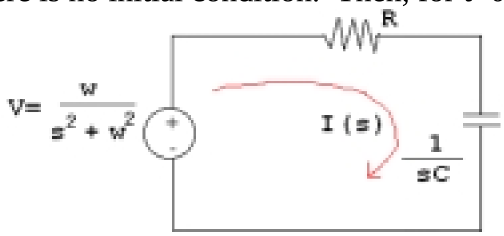
$$I(s)=\frac{V(s)}{R+1/sC}=\frac{\frac{1}{s+a}\cdot sC}{RCs+1}=\frac{sC}{(RCs+1)(s+a)}=\frac{s/R}{(s+a)(s+1/RC)}=\frac{A}{s+a}+\frac{B}{s+1/RC}$$

For, *i*(t), we apply Inverse Laplace Transformation:

2. Sinusoidal Source Case



Similarly, we assume that there is no initial condition. Then, for t>0, the s-domain circuit is:



The s-domain equation for I(s):

$$I(s) = \frac{V}{R + 1/sC} = \frac{\dfrac{w}{s^2 + w^2}}{R + 1/sC} = \frac{sC \cdot \dfrac{w}{s^2 + w^2}}{RCs + 1} = \frac{ws/R}{(s^2 + w^2)(s + 1/RC)}$$

By partial expansion, we have:

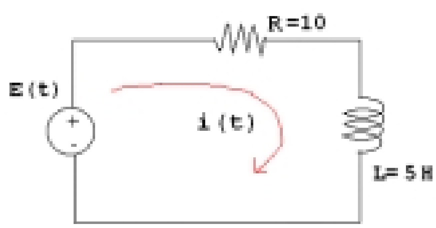
$$I(s) = \frac{ws/R}{(s^2 + w^2)(s + 1/RC)} = \frac{A}{(s + 1/RC)} + \frac{Bs + D}{(s^2 + w^2)} = \frac{A}{(s + 1/RC)} + \frac{Bs}{(s^2 + w^2)} + \frac{\dfrac{D}{w} \cdot w}{(s^2 + w^2)}$$

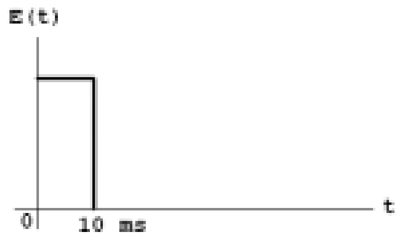
Therefore, i(t) becomes:

____

3. Pulsed Source Case

RL circuit below is excited at t=0 by a narrow voltage pulse whose magnitude is 1000 V and whose width is 10 ms as shown below. We want to solve for i(t) using s-domain analysis.

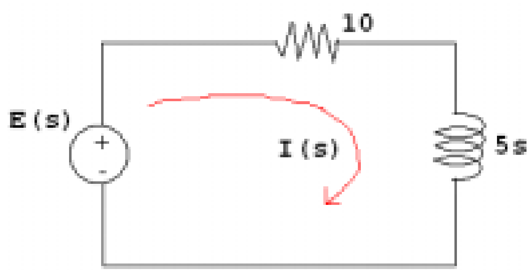




Since there is no initial condition, we can draw s-domain circuit.
(a) **What is time-domain source equation, E(t)?**

(b) **What is the s-domain source, E(s)?**

(c) Then, the s-domain circuit becomes:

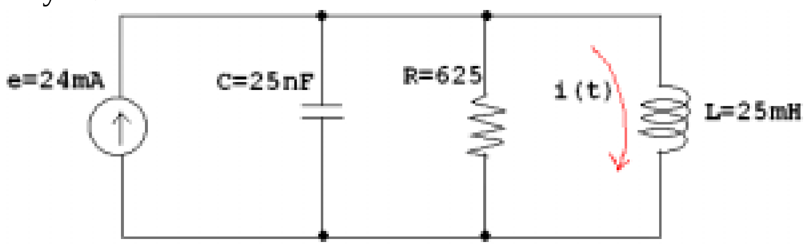


(d) **Write the equation for I(s):**

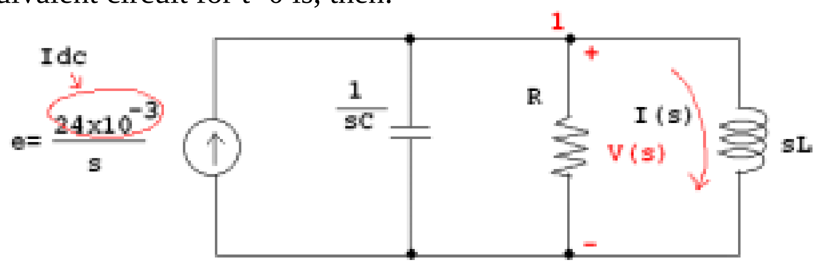
(e) **Then, what is *i*(t)?** (cf: $L\{f(t-a)u(t-a)\} = e^{-as}F(s)$)

4. Response of Parallel RLC case

Assuming that there is no initial condition, we will find the current through the inductor, *i*(t), using s-domain analysis.



The s-domain equivalent circuit for t>0 is, then:



Additionally, we defined a node (1) and it's node voltage as V(s).

(a)From the circuit, we have the following relationship for the current: $I(s) = \frac{V(s)}{sL}$

Therefore, we need an equation of V(s).

(b)Write a node voltage equation.

____

(c)Then, write the equation for I(s):

____

(d) Let's, plug in the values for the elements and the source:

____

(e) Examination of the roots: one real root and complex roots.
So we have to have the following partial expansion.

$$I(s) = \frac{A}{s} + \frac{Bs + C}{(s^2 + 64000s + 16\times10^8)}$$

(f) By residue method, we can get easily for A: $A = \frac{384\times10^5}{16\times10^8} = 24\times10^{-3}$

(g) By algebraic Method we get B and C: B=-24×10^{-3}, C=-1536

(h) Finally,

$$I(s) = 24\times10^{-3}\{\frac{1}{s} - \frac{s+64000}{(s^2+64000s+16\times10^8)}\}$$

$$= 24\times10^{-3}\{\frac{1}{s} - \frac{s+64000}{(s+32000)^2+24000^2}\}$$

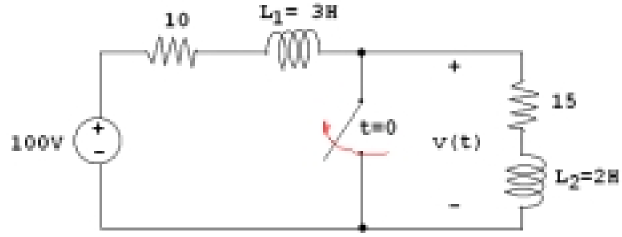
$$= 24\times10^{-3}\{\frac{1}{s} - \frac{s+32000}{(s+32000)^2+24000^2} - \frac{(4/3)\cdot24000}{(s+32000)^2+24000^2}\}$$

(i) Therefore, by the Inverse Laplace Transform, we have:

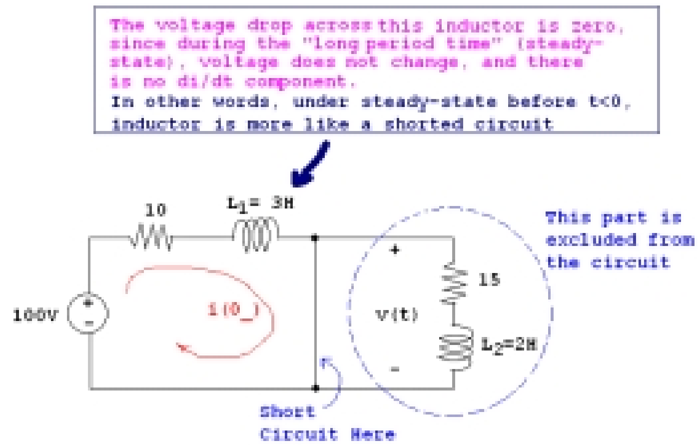
---

5. Switching Operation and Impulse Response Case

Now Let's consider a case which has impulse function in its response.
The switch in the circuit below has been in the closed position for a long time, and at t=0, it opens. We want to find the voltage v(t) for t>0.

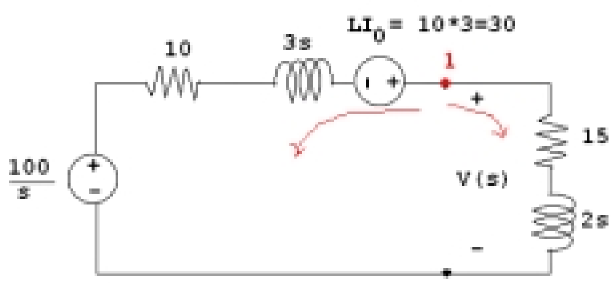


(a) The circuit just before t=0.



Therefore the initial current through the inductor L_1 is: $I_{10} = \dfrac{100}{10} = 10\,[\mathrm{A}]$

This initial current must be reflected in the s-domain circuit, for t>0
(b) s-domain circuit after t>0.



(c) Write s-domain equation for V(s) using node-voltage equation at node 1.

____

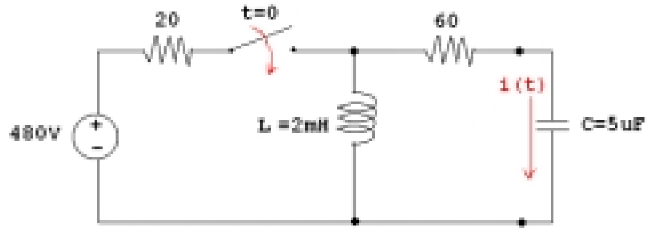
(d) Then, what is the voltage *v*(t)?

____

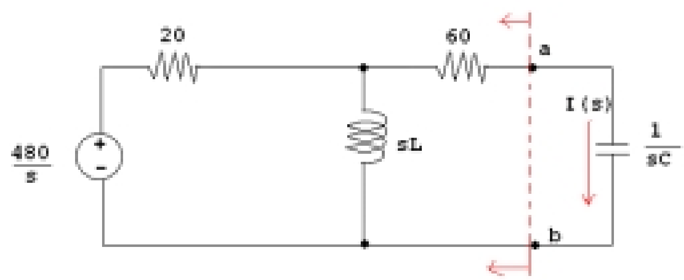
6. Thevenin Equivalent Circuit in s-domain environment

The same principle of Thevenin Equivalent Circuit we discussed in time-domain can be applied to s-domain analysis. With an example, we display the Thevenin equivalent circuit approach in circuit analysis. Remember, though, this approach is just a method of many different approaches you can apply for circuit analysis. In other words, most of the time, simple node voltage method is more than enough.
In this discussion, however, we apply Thevenin circuit just for illustration.

Let's find the current through the capacitor in the circuit below by applying Thevenin equivalent circuit. There is no initial condition before time t=0.



(a) The s-domain circuit is:



(b) And, we will find the equivalent circuit of that of left side of the terminals **a** and **b**.
(c) The Thevenin voltage at **a** and **b** is the open circuit voltage at **a** and **b**:
Write the open circuit voltage equation here? (voltage divider)

____

(d) For Thevenin Impedance, Z_{th}, we can apply the 'equivalent resistance method.'
Write the Thevenin Impedance Equation here? (source deactivation)

____

(e) Therefore, we have the simplified circuit as shown below.

(f) Now then, we can draw equation for the current I(s).
Write the equation for I(s) here.

____

(g) Finally, the current in time domain is:

____

(h) How about the voltage across the capacitor, v(t)?

____