

**Class Note 10: Node Voltage Analysis with Voltage Sources between two nodes - who cares if they call it “super-node” case?**

### Background:

When a voltage source (independent or dependent) is directly connected between two essential nodes, there is no way to directly express the element current through the voltage source for your node voltage equation. In such case, the two nodes connecting the voltage source could be reduced to a single node, called a *supernode*. This is what “they” say about the case.

However, my approach does not recommend you find and use the 'super node,' instead, I recommend you use the same old node voltage method whatever element you have between two nodes. But with something called "one more step." This method works perfect under any condition. Remember, “super-sizing” even nodes may be harmful to your health.

### McSupersizes to be phased out

Wednesday, March 3, 2004 Posted: 2:15 AM EST (07:15 GMT)

CHICAGO, Illinois (AP) -- Say goodbye to those supersized fries -- McDonald's is slimming down its menu.

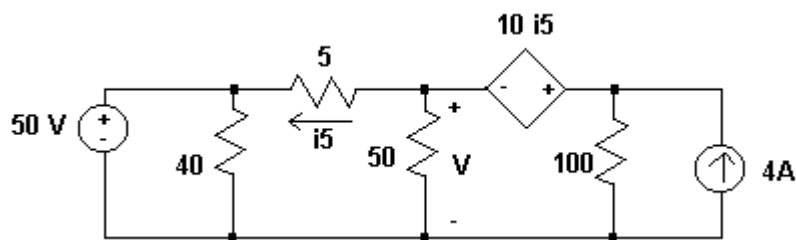


The next discussion explains what I mean by the regular node voltage method with "one more step."

### Example Problem 1:

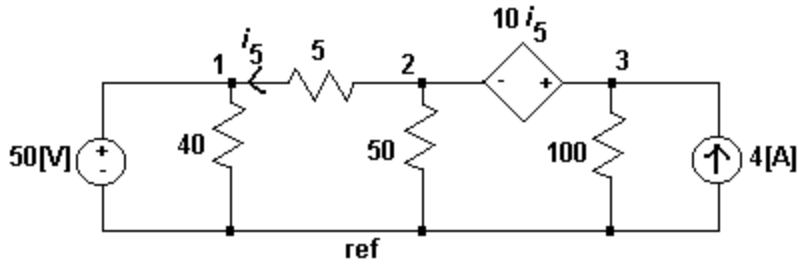
**Q:** From the circuit below, find the voltage  $V$ .

Do you have a dependent voltage source between two essential nodes?

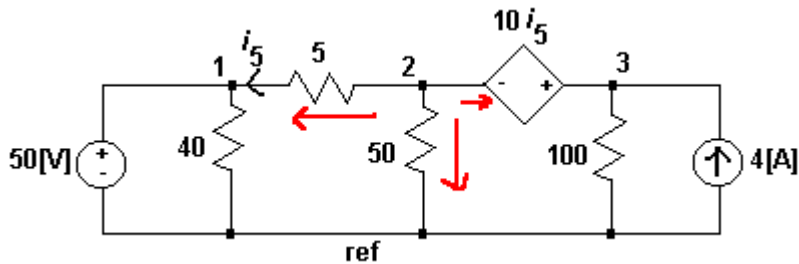


### Solution

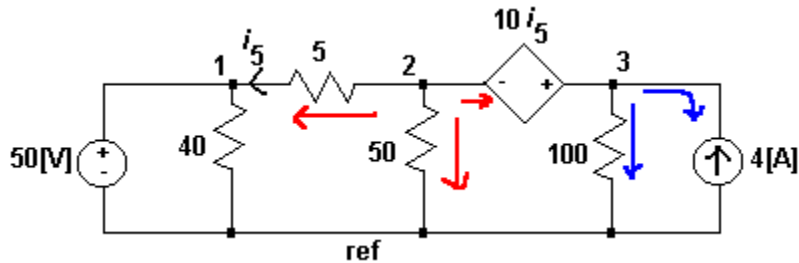
1. We quickly found that we have 4 essential nodes. The bottom one is picked as the reference. So we mark the nodes as below. Then the question is to find the voltage  $V_2$ .



2. Since we already know  $V_1=50$  V by the source. We do not need equation at node 1.
3. At the node 2, we can easily express the current to the left [i.e.,  $\frac{V_2 - 50}{5}$ ] and the current down to the reference [i.e.,  $\frac{V_2}{50}$ ] easily. But how do we express the current to the right?



4. Remember that node-voltage method is KCL in disguise? Here the "one more step" enters in. Do you see that the current to the right (at node 2) is the sum of the current at the next node (i.e., node 3)?



5. Then the current to the right at node 2 becomes:  $\left[ \frac{V_3}{100} - 4 \right]$
6. Now we can write a node voltage equation at node 2 as follows:

$$\frac{V_2 - 50}{5} + \frac{V_2}{50} + \left[ \frac{V_3}{100} - 4 \right] = 0 \text{ -----(1)}$$

$\leftarrow$        $\downarrow$        $\rightarrow$

You may be surprised by what equation (1) suggests. The equation (1) makes the nodes 2 and 3 together as a single node! Do you see it? This combined (in the equation) node is called a *supernode*. But it's not a magic, but a result from the "one more step" approach. Remember, we got this "super node" equation without using the term "super node."

7. Since, the equation (1) involves nodes 2 and 3, we do not need another node voltage equation at node 3. If you try anyway, then you would reach at the same equation. (See the bottom part of page 3 if you want to prove this point)

8. So all nodes are used in deriving Equation (1), and the equation has 2 variables. This means we have draw one more equation to solve the problem.

9. Since the dependent voltage source is connected between node  $a$  and  $b$ , that voltage is nothing but the voltage between the two nodes, i.e.,  $10i_5 = V_3 - V_2$

10. Also, form the circuit, current  $i_5$  can be expressed by:  $i_5 = \frac{V_2 - 50}{5}$

11. By the substitution of  $i_5$ , equation in 9 above becomes:

$$10i_5 = V_3 - V_2 \Rightarrow 2(V_2 - 50) = V_3 - V_2 \Rightarrow V_3 = 3V_2 - 100 \text{ ----(2)}$$

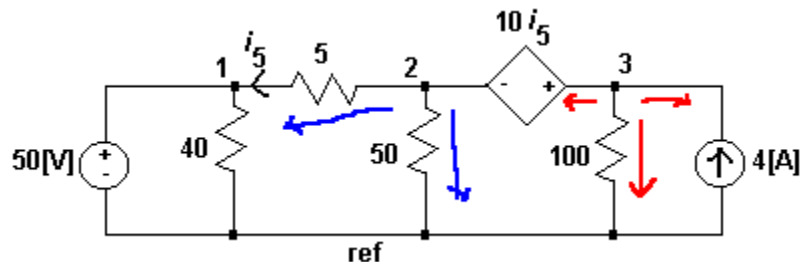
12. By plugging equation (2) to equation (1):

$$\frac{V_2 - 50}{5} + \frac{V_2}{50} + \left[ \frac{V_3}{100} - 4 \right] = 0 \Rightarrow \frac{V_2 - 50}{5} + \frac{V_2}{50} + \left[ \frac{3V_2 - 100}{100} - 4 \right] = 0 \Rightarrow 25V_2 = 1500$$

----->  $V_2 = 60$  (and additionally,  $V_3 = 80$ )

**Proof of the argument in 7 above:**

At node 3, with the same approach of "one more step" we can easily get the expression of the current to the left at node 3.

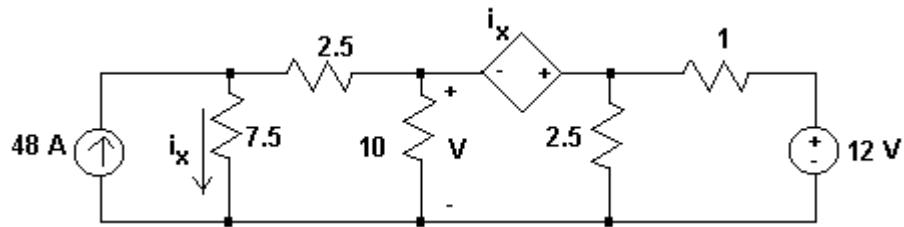


$$-4 + \frac{V_3}{100} + \left[ \frac{V_2 - 50}{5} + \frac{V_2}{50} \right] = 0 \text{ -----> } \frac{V_2 - 50}{5} + \frac{V_2}{50} + \left[ \frac{V_3}{100} - 4 \right] = 0$$

### Example Problem 2:

Another case of a dependent voltage source between two nodes.

Q: Find the voltage  $V$ .

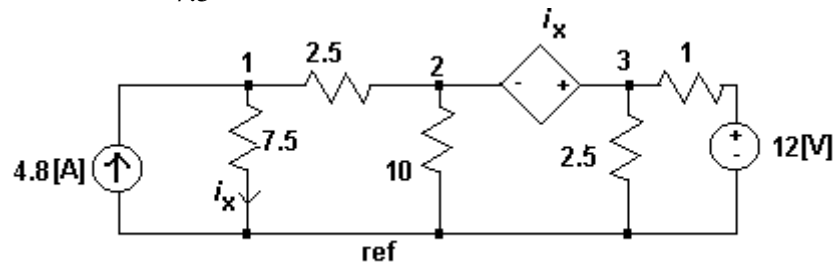


### Solution:

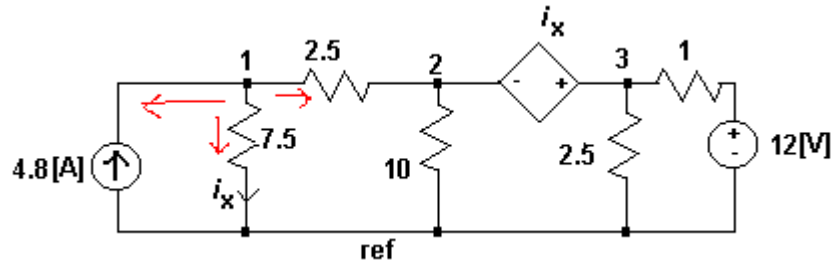
1. Since we have 4 essential nodes we mark them

2. We picked the bottom one as the reference node as shown below. Then the question is to find

$V_2$ . (Also, we see that  $i_x = \frac{V_1}{7.5}$ ).

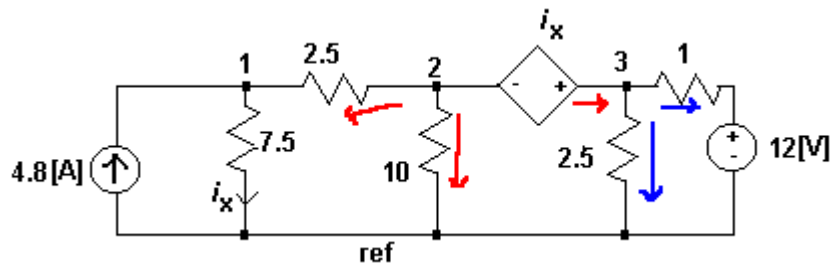


3. Let's apply KCL at node 1:  $-4.8 + \frac{V_1}{7.5} + \frac{V_1 - V_2}{2.5} = 0 \rightarrow 4V_1 - 3V_2 = 36$  -----(1)



4. KCL at node 2: (We use "one more step" approach)

$$\frac{V_2 - V_1}{2.5} + \frac{V_2}{10} + \left[ \frac{V_3}{2.5} + \frac{V_3 - 12}{1} \right] = 0 \rightarrow -4V_1 + 5V_2 + 14V_3 = 120$$
 -----(2)



5. Since node 3 is already included in equation (2), all nodes are covered but we have only 2 equations with 3 variables. Apparently we need one more equation.

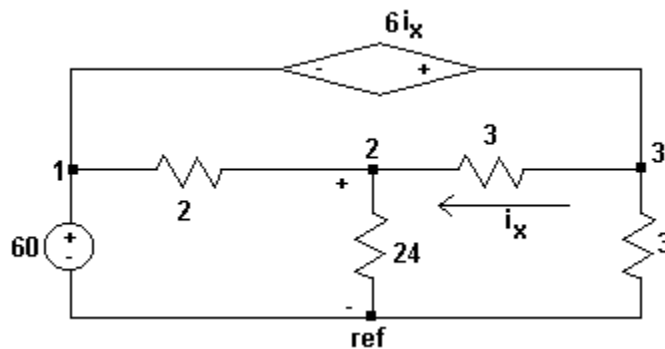
6. As we did in Example Problem 1, we express the dependent voltage and the voltage difference between nodes 2 and 3:  $i_x = V_3 - V_2 = \frac{V_1}{7.5} \rightarrow 4V_1 + 30V_2 - 30V_3 = 0$  -----(3)

7. By plugging equation (3) to (1) and (2), we have:  $V_1 = 15$ ,  $V_2 = 8$ , and  $V_3 = 10$

8. Then,  $i_x = \frac{V_1}{7.5} = 2$  [A]

**Example Problem 3:**

**Q:** Find  $V_2$ .



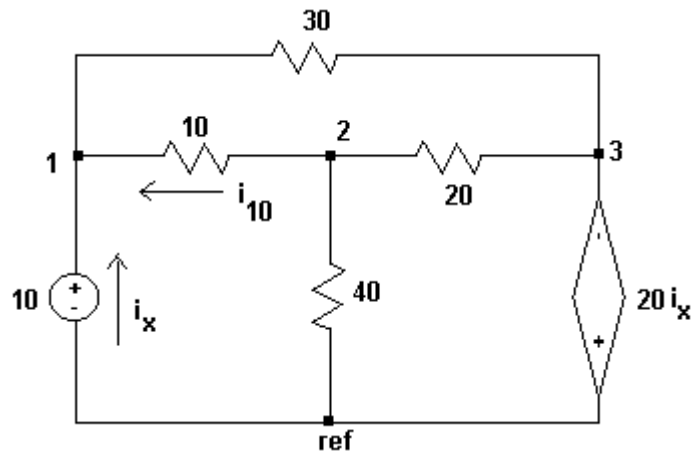
As you see above, we already have all the marks for nodes and the reference.

**Solution**

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**Example Problem 4:**

Q: Using node-voltage method, find the current  $i_{10}$ .



**SOLUTION:**

Step 1: Essential nodes

Step 2: Reference

Step 3: Node voltage equations.