Class Note 07: Δ↔Y Transformation Example

Problem Setting:
Find currents i_0, i_1, and i_2 of the circuit below.

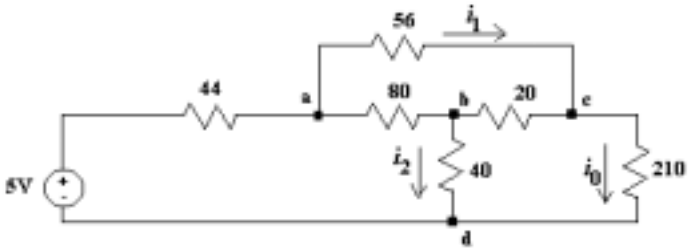

Fig 1

SOLUTION

1. Let's observe the circuit before we make any move. Do you see Y and Δ resistors?
Yes, we see them as shown below. But remember, in most of circuit problems, it's an one way traffic as far as the transformation direction concerns: $\Delta \rightarrow$ Y.

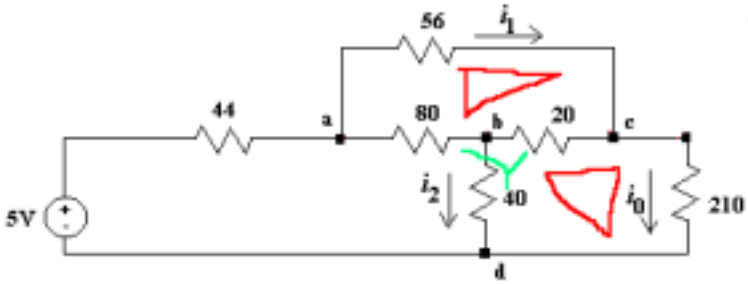

Fig 2

2. If you convert the Δ resistors (right side) to Y then it looks like this: *Why not the top Δ?*

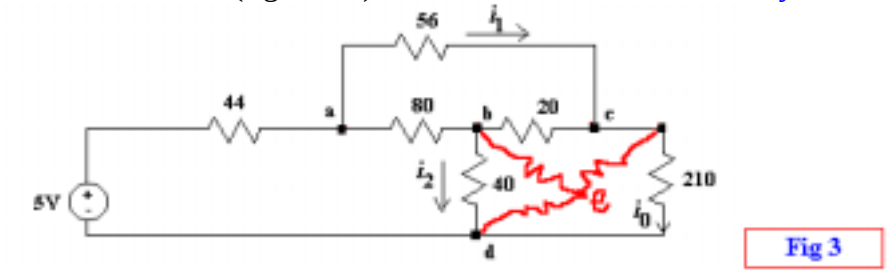

Fig 3

Here we have to remember that: $i_2 = \frac{V_{bd}}{40}$ and $i_0 = \frac{V_{cd}}{210}$; i_1 is the current through 56Ω resistor

3. Now we have the following simplified circuit:

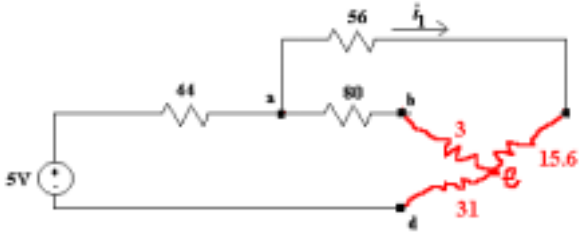

Fig 4

4. It further simplifies to:

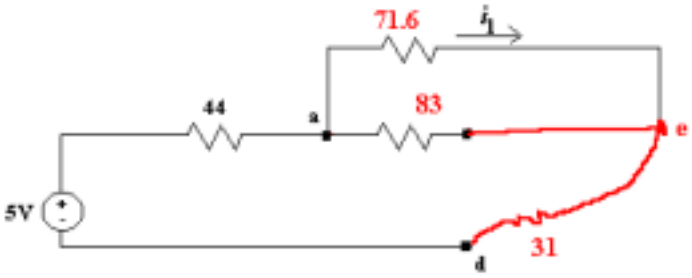

Fig 5

4. Finally, the circuit is reduced to a single loop one:

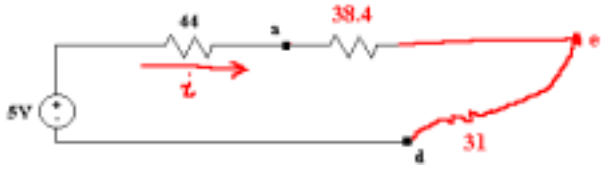

Fig 6

5. Then the current flowing through the resistors is: $i = \dfrac{5}{(44+38.4+31)} = 0.044\,[\text{A}]$ (Fig.6)

6. Now let's find currents by the following steps: (it looks like a wave propagation, or, stating from an open cell, marking mines or opening neighboring cells, in your famous minesweeper game. Or do you feel it?)

(1) The voltage $V_{ae} = 0.044 \times 38.4 = 1.69\,[\text{V}]$ (Fig.6)

(2) Therefore the current $i_1 = \dfrac{V_{ae}}{71.6} = 0.0236\,[\text{A}]$ (Fig.5)

(3) The current through the 80 resistor is: $i_{80} = \dfrac{V_{ae}}{83} = 0.02036\,[\text{A}]$ (Fig. 4)

(4) Therefore, voltage $V_{ab} = i_{80} \times 80 = 1.63\,[\text{V}]$ (Fig.4 and Fig.3)

(5) Also, voltage $V_{ad} = i \times (38.4+31) = (0.044)(69.4) = 3.0536\,[\text{V}]$ (Fig. 6)

(6) Then, from above two voltages, we have voltage:

$$V_{bd} = V_{ad} - V_{ab} = 3.0536 - 1.63 = 1.4236\,[\text{V}] \text{ (Fig. 3)}$$

(7) Then, current $i_2 = \dfrac{V_{bd}}{40} = \dfrac{1.4236}{40} = 0.0356\,[\text{A}]$ (Fig. 3)

(8) By KCL at node *b*, we get the current through 20 resistor (toward right):

$$i_{20} = i_{80} - i_2 = -0.01524\,[\text{A}] \text{ (Fig. 2)}$$

(9) Therefore, $i_0 = i_1 + i_{20} = 0.0236 + (-0.01524) = 0.00836\,[\text{A}]$ (Fig. 2)